Ecole de Recherche CIMPA & Workshop

http://cimpa2020fpn.ump.ma

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Equations aux dérivées partielles non-linéaires, théorie spectrale et applications

Cours :

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Titre : Eigenvalues, eigenvectors, and solutions of differential and difference equations on networks

Résumé : The mini-course will begin with an introduction to models of scientific phenomena on both discrete and metric graphs. Models related to quantum theory to be discussed will include the graph Laplacian, magnetic graph Laplacians, quantum graphs, and quantum random walks. A selection of models from other areas will also be discussed, chosen from areas such as quantum information and diffusion on networks, some of which may be nonlinear. We will ask about both the similarities and the sometimes major differences between how models work on networks versus those on domains or manifolds.

Next, the connections between eigenvalues and the structure of the graph will be reviewed both through theorems and examples. General relations such as sum rules and Hardy inequalities will be developed and adapted to graphs. Particular attention will be paid to the extreme cases, which are often unique. Variational and perturbation methods and notions of compactness will be developed.

After that we will describe the analysis of special solutions of equations on networks, especially eigenvectors and eigenfunctions, and why they are important for science. Necessary background from Hilbert space theory and from ordinary and partial differential equations will be discussed as needed. Topics to be covered in this section will include maximum principles and Harnack inequalities.

We will review what is known about localization of eigenfunctions, the numbers of nodal domains, and semiclassical estimates. The notion of a "landscape function" will be introduced and used to create explicit, adaptive estimates of eigenfunctions. Different methods will be brought in from the theories of ordinary and partial differential equations, which have advantages in different circumstances.

At the end of the mini-course topics will be selected to match the interests of the students, whether in quantum theory, such as quantum ergodicity or resonances; or in data science, such as community detection; or phenomena that arise in nonlinear dynamics on graphs.